

Angles

Find the measures of the angles.

<p>1 _____ = <math>m\angle 1</math></p> <p>2 _____ = <math>m\angle 2</math></p>		<p>3 _____ = <math>m\angle 3</math></p> <p>4 _____ = <math>m\angle 4</math></p>
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<p>5 _____ = <math>m\angle AXB</math></p> <p>6 _____ = <math>m\angle FXB</math></p> <p>7 _____ = <math>m\angle FXE</math></p> <p>8 _____ = <math>m\angle CXD</math></p> <p>9 _____ = <math>m\angle DXE</math></p> <p>10 _____ = <math>m\angle CXE</math></p>		<p>11 _____ = <math>m\angle FXC</math></p> <p>12 _____ = <math>m\angle FXD</math></p> <p>13 _____ = <math>m\angle AXD</math></p> <p>14 _____ = <math>m\angle AXE</math></p> <p>15 _____ = <math>m\angle BXD</math></p> <p>16 _____ = <math>m\angle AXC</math></p>
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<p>17 _____ = <math>m\angle 1</math></p> <p>18 _____ = <math>m\angle 2</math></p> <p>19 _____ = <math>m\angle 3</math></p>		<p>20 _____ = <math>m\angle 4</math></p> <p>21 _____ = <math>m\angle 5</math></p> <p>22 _____ = <math>m\angle 6</math></p>
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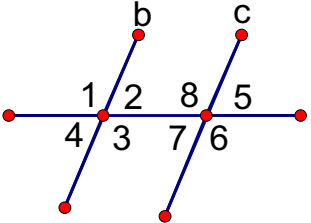
<p><math>m\angle AEB = 5x + 5</math>; <math>m\angle BEC = 15x + 15</math></p> <p>23 _____ = <math>x</math></p> <p>24 _____ = <math>m\angle BEC</math></p> <p>25 _____ = <math>m\angle AED</math></p> <p>26 _____ = <math>m\angle AEB</math></p> <p>27 _____ = <math>m\angle DEC</math></p>		<p><math>m\angle AEB = 3x + 20</math>; <math>m\angle CED = 5x - 10</math></p> <p>28 _____ = <math>x</math></p> <p>29 _____ = <math>m\angle BEC</math></p> <p>30 _____ = <math>m\angle AED</math></p> <p>31 _____ = <math>m\angle AEB</math></p> <p>32 _____ = <math>m\angle DEC</math></p>
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Perpendicular and Parallel Lines

If a transversal intersects two PARALLEL lines:

- \* then the alternate interior angles are CONGRUENT.
- \* then the alternate exterior angles are CONGRUENT.
- \* then the corresponding angles are CONGRUENT.
- \* then the same-side interior angles are SUPPLEMENTARY.  $m\angle 2 + m\angle 8 = 180$
- \* and it is perpendicular to one line, then it is perpendicular to the other line also.

$b \parallel c$   
 $\angle 2 \cong \angle 7$   
 $\angle 1 \cong \angle 6$   
 $\angle 4 \cong \angle 3$

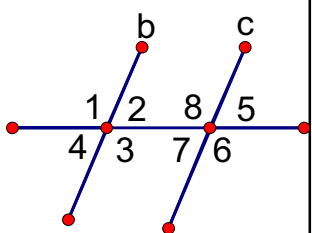


Find all the remaining angles from the given angle measure.

$b \parallel c$

	$m\angle 1 =$	$m\angle 2 =$	$m\angle 3 =$	$m\angle 4 =$	$m\angle 5 =$	$m\angle 6 =$	$m\angle 7 =$	$m\angle 8 =$
1	150°							
2							70°	
3				65°				
4		$x^\circ$						

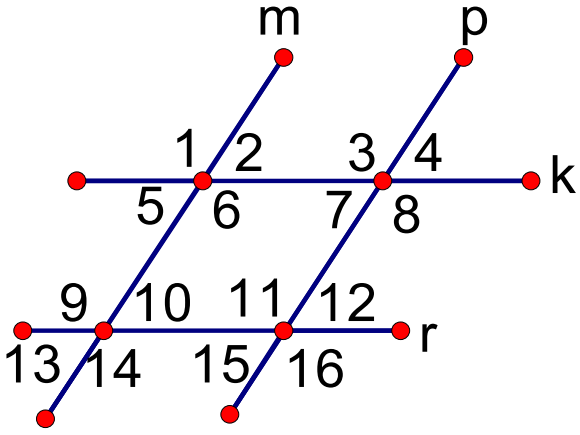
5  $\angle 1 \cong \angle \underline{\quad} \cong \angle \underline{\quad} \cong \angle \underline{\quad}$       6  $\angle 1$  is supplementary with  $\angle \underline{\quad}$ ,  $\angle \underline{\quad}$ ,  $\angle \underline{\quad}$  &  $\angle \underline{\quad}$ .



Find the angle measure with the given information.

$m \parallel p \text{ \& } k \parallel r$

7 $m\angle 2 = 80^\circ$	$m\angle 12 =$ _____
8 $m\angle 15 = 78^\circ$	$m\angle 2 =$ _____
9 $m\angle 8 = 110^\circ$	$m\angle 5 =$ _____
10 $m\angle 13 = 84^\circ$	$m\angle 12 =$ _____
11 $m\angle 5 = 86^\circ$	$m\angle 16 =$ _____
12 $m\angle 3 = 108^\circ$	$m\angle 13 =$ _____
13 $m\angle 1 = 110^\circ$	$m\angle 12 =$ _____
14 $m\angle 7 = 88^\circ$	$m\angle 9 =$ _____
15 $m\angle 6 + m\angle 14 = 190^\circ$	$m\angle 3 =$ _____
16 $m\angle 12 + m\angle 15 = 150^\circ$	$m\angle 5 =$ _____
17 $m\angle 9 + m\angle 14 = 180^\circ$	$m\angle 4 =$ _____
18 $m\angle 5 + m\angle 7 = 158^\circ$	$m\angle 12 =$ _____



Use the figure above to fill in the boxes with angles congruent to the given angle.

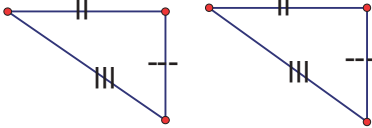
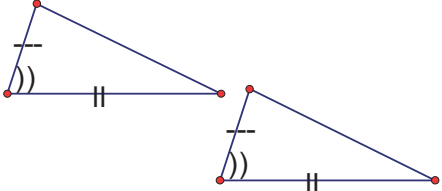
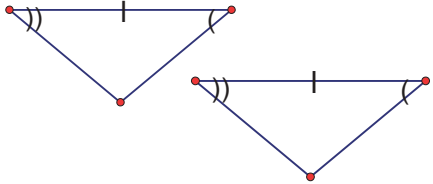
19  $\angle 1 \cong$

20  $\angle 2 \cong$

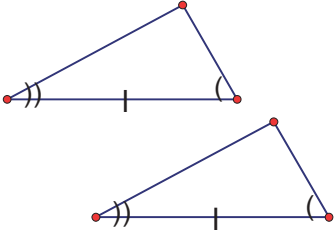
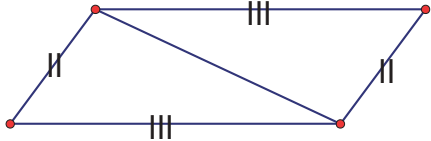
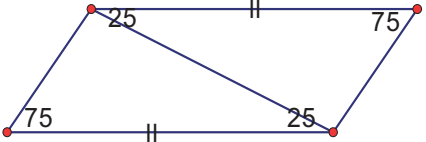
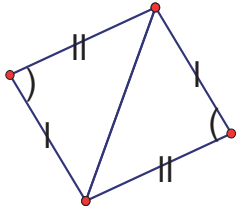
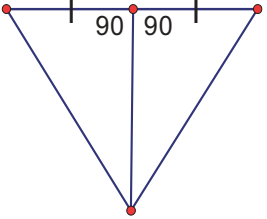
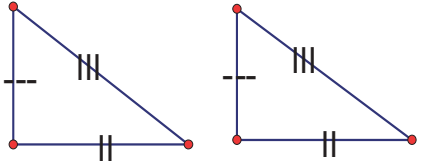
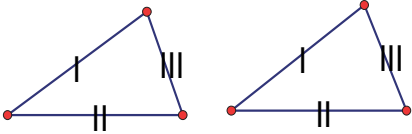
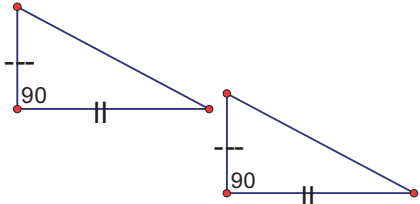
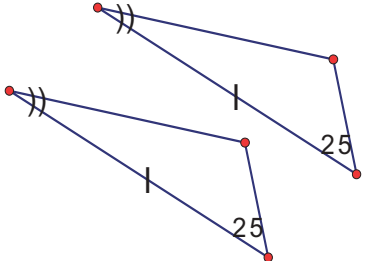
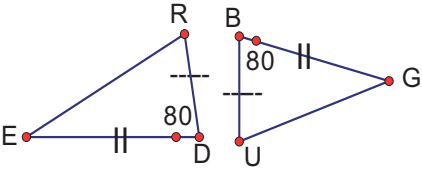
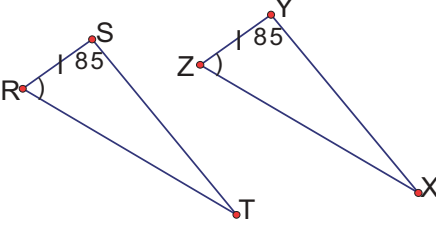
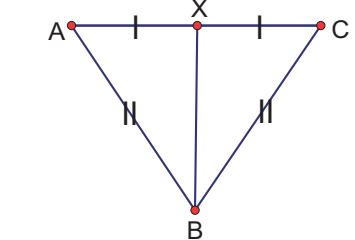
Congruent Triangles

Congruent Triangles

Ways to Prove Triangles are Congruent:

<p style="text-align: center;"><b>SSS</b> side-side-side</p>  <p style="text-align: center;"><u>All corresponding sides</u> are congruent.</p>	<p style="text-align: center;"><b>SAS</b> side-angle-side</p>  <p style="text-align: center;"><u>2 corr. sides</u> and the <u>angle between</u> are congruent.</p>	<p style="text-align: center;"><b>ASA</b> angle-side-angle</p>  <p style="text-align: center;"><u>2 corr. angles</u> and the <u>side between</u> are congruent.</p>
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Name the postulate: SSS, SAS, or ASA that proves the triangles congruent.

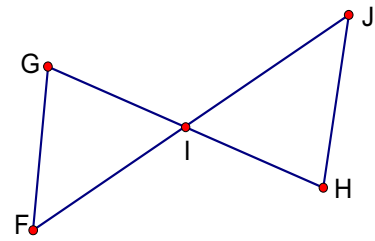
<p>1</p> 	<p>5</p>  <p>*Hint: A shared side is considered a congruent corresponding side.</p>	<p>9</p> 
<p>2</p> 	<p>6</p> 	<p>10</p> 
<p>3</p> 	<p>7</p> 	<p>11</p> 
<p>4 Write the congruence statement.</p> <p><math>\Delta \_ \cong \Delta \_</math></p> 	<p>8 Write the congruence statement.</p> <p><math>\Delta \_ \cong \Delta \_</math></p> 	<p>12 Write the congruence statement.</p> <p><math>\Delta \_ \cong \Delta \_</math></p> 

### Congruent Triangles

#### Proofs

1) Given: GH and FJ bisect each other.

Prove:  $\triangle FGI \cong \triangle JHI$



STATEMENTS

REASONS

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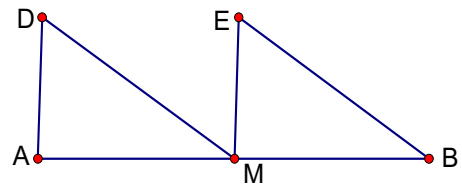
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2) Given: M is the midpoint of AB; AD  $\parallel$  ME; DM  $\parallel$  EB

Prove: DM  $\cong$  EB



STATEMENTS

REASONS

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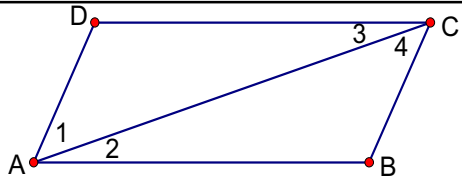
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3) Given: AB  $\parallel$  DC; AB  $\cong$  DC

Prove:  $\angle B \cong \angle D$



STATEMENTS

REASONS

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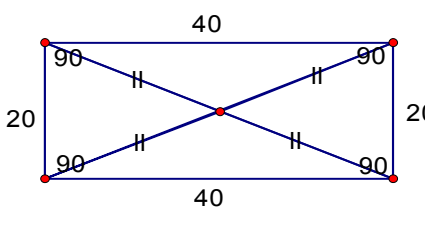
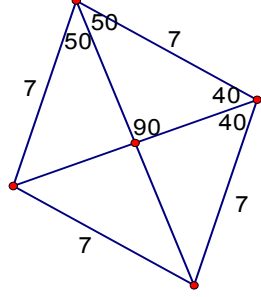
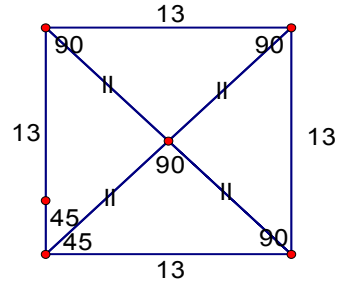
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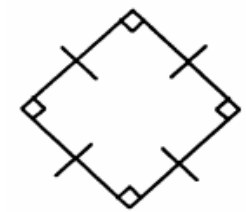
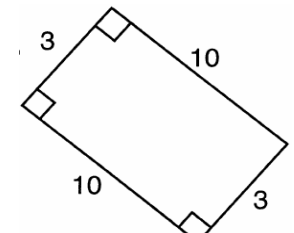
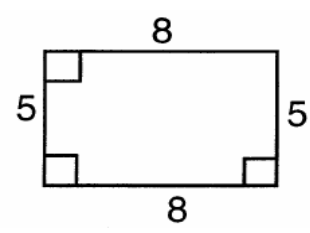
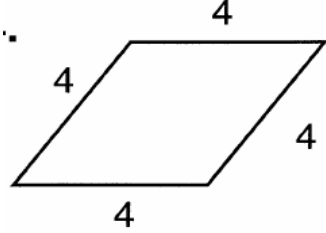
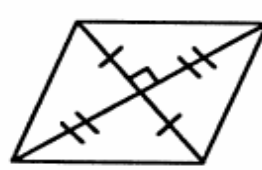
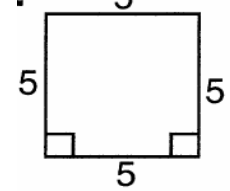
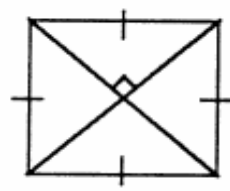
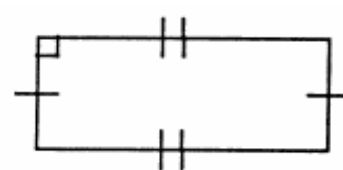
Quadrilaterals I

Special Parallelograms

All three of these have all the properties of a parallelogram also.

Rectangle	Rhombus	Square
		
<ul style="list-style-type: none"> <li>• It has all right angles.</li> <li>• Diagonals are congruent.</li> </ul>	<ul style="list-style-type: none"> <li>• All sides are congruent.</li> <li>• Diagonals are perpendicular.</li> <li>• Diagonals bisect opposite angles.</li> </ul>	<ul style="list-style-type: none"> <li>• All sides are congruent.</li> <li>• It has all right angles.</li> <li>• Diagonals are congruent.</li> <li>• Diagonals are perpendicular.</li> <li>• Diagonals bisect opposite angles.</li> </ul>

Classify the following parallelograms as a rectangle, rhombus, or square.

<p>1</p> 	<p>5</p> 
<p>2</p> 	<p>6</p> 
<p>3</p> 	<p>7</p> 
<p>4</p> 	<p>8</p> 

Quadrilaterals I

ABCD is a rectangle.  $AB = 12$ ;  $BC = 9$ ;  $m\angle 1 = 60^\circ$

1	DC =	_____		6	$m\angle 2 =$	_____
2	AD =	_____		7	DB =	_____
3	AC =	_____		8	DX =	_____
4	AX =	_____		9	$m\angle 3 =$	_____
5	$m\angle ADC =$	_____		10	$m\angle 4 =$	_____

You will need to use pythagorean theorem:  $a^2 + b^2 = c^2$ .

KITE is a rhombus.  $KI = 5$ ;  $m\angle ITE = 60^\circ$ ;  $KT = 8$

11	ET =	_____		16	$m\angle KIT =$	_____
12	$m\angle EKI =$	_____		17	$m\angle 7 =$	_____
13	$m\angle 1 =$	_____		18	SI =	_____
14	$m\angle KSE =$	_____		19	$m\angle 4 =$	_____
15	KS =	_____		20	EI =	_____

You will need to use pythagorean theorem:  $a^2 + b^2 = c^2$ .

ABCD is a square.  $CD = 10$

21	AB =	_____		26	BD =	_____
22	$m\angle ABC =$	_____		27	$m\angle CXD =$	_____
23	BC =	_____		28	AC =	_____
24	$m\angle BCD =$	_____		29	$m\angle 4 =$	_____
25	$m\angle 1 =$	_____		30	XD =	_____

You will need to use pythagorean theorem:  $a^2 + b^2 = c^2$ .

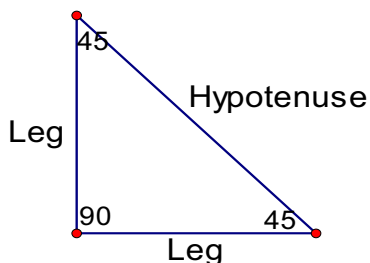


Right Triangles

Special Right Triangles

These two triangles have special relationships among their sides. Memorize the formulas for the class.

45° - 45° - 90° Triangle

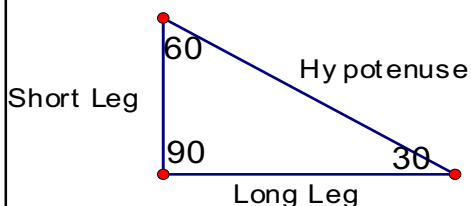


Since a 45-45-90 is an isosceles triangle, the legs are congruent. If a right triangle is marked with its acute angles congruent, this is a 45-45-90 triangle.

Leg to Hypotenuse: multiply the number by  $\sqrt{2}$   
 Hypotenuse to Leg: divide the number by  $\sqrt{2}$ .

$$H = L\sqrt{2}$$

30° - 60° - 90° Triangle



The short leg (S) is across from the 30° angle.  
 The long leg (L) is across from the 60° angle.

Short Leg to Hypotenuse: multiply the number by 2  
 Short Leg to Long Leg: multiply the number by  $\sqrt{3}$

Hypotenuse to Short Leg: divide the number by 2  
 Long Leg to Short Leg: divide the number by  $\sqrt{3}$

Always try to find the short leg first!

$$L = S\sqrt{3}$$

$$H = 2S$$

Solve for the variables.

<p>1</p> <p>a = _____</p> <p>b = _____</p>	<p>4</p> <p>a = _____</p> <p>b = _____</p>	<p>7</p> <p>a = _____</p> <p>b = _____</p>
<p>2</p> <p>a = _____</p> <p>b = _____</p>	<p>5</p> <p>a = _____</p> <p>b = _____</p>	<p>8</p> <p>a = _____</p> <p>b = _____</p>
<p>3</p> <p>a = _____</p> <p>b = _____</p>	<p>6</p> <p>a = _____</p> <p>b = _____</p>	<p>9</p> <p>a = _____</p> <p>b = _____</p>

# Right Triangle Trigonometry

## Right Triangle Trigonometry

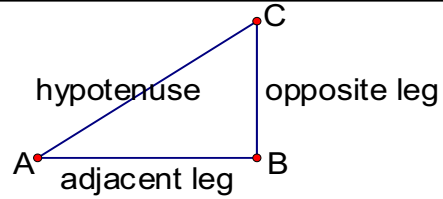
### Tangent Ratio

tangent of  $\angle A = \frac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A}$

also seen as:

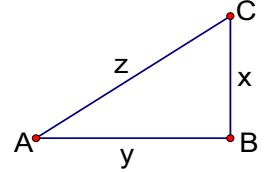
$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

Each acute angle in the triangle has its own tangent ratio.



For this triangle:  $\tan A = \frac{x}{y} = \frac{\text{opposite}}{\text{adjacent}}$

The legs switch for the other acute angle.  $\tan C = \frac{y}{x} = \frac{\text{opposite}}{\text{adjacent}}$



Find the tangent ratio for each acute angle.

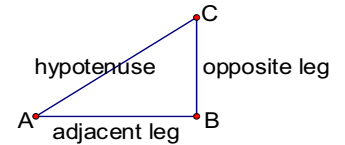
<p>1</p> <p><math>\tan A = 5/12</math></p> <p><math>\tan B =</math></p>	<p>3</p> <p><math>\tan A =</math></p> <p><math>\tan B =</math></p>
<p>2</p> <p><math>\tan A =</math></p> <p><math>\tan B =</math></p>	<p>4</p> <p><math>\tan R =</math></p> <p><math>\tan S =</math></p>

### Sine & Cosine Ratios

sine ratio =  $\frac{\text{opposite leg}}{\text{hypotenuse}}$

cosine ratio =  $\frac{\text{adjacent leg}}{\text{hypotenuse}}$

Relative to  $\angle A$ :

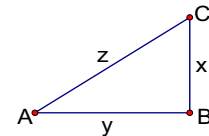


$$\sin A = x/z$$

$$\sin C = y/z$$

$$\cos A = y/z$$

$$\cos C = x/z$$



Find the sine & cosine ratios for each acute angle.

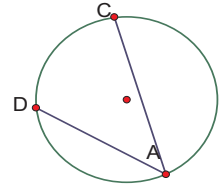
<p>5</p> <p><math>\sin A = 5/13</math></p> <p><math>\sin B =</math></p> <p><math>\cos A =</math></p> <p><math>\cos B =</math></p>	<p>7</p> <p><math>\sin A =</math></p> <p><math>\sin B =</math></p> <p><math>\cos A =</math></p> <p><math>\cos B =</math></p>
<p>6</p> <p><math>\sin A =</math></p> <p><math>\sin B =</math></p> <p><math>\cos A =</math></p> <p><math>\cos B =</math></p>	<p>8</p> <p><math>\sin R =</math></p> <p><math>\sin S =</math></p> <p><math>\cos R =</math></p> <p><math>\cos S =</math></p>

Circles I

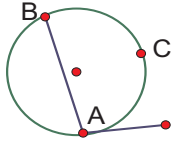
Inscribed angles: its vertex is on a circle and its sides are chords of the circle  
 The measure of an inscribed angle is half the measure of its intercepted arc.

$$m\angle A = \frac{1}{2}m\widehat{CD}$$

ex: If  $m\angle A = 30^\circ$ , then  $m\widehat{CD} = 60^\circ$ .



Also: the measure of an angle formed from a chord and a tangent is half its intercepted arc

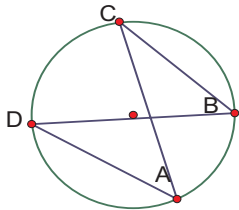


$$m\angle A = \frac{1}{2}m\widehat{BCA}$$

ex: If  $m\widehat{BCA} = 150^\circ$ , then  $m\angle A = 75^\circ$ .

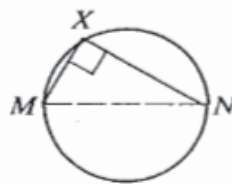
If 2 inscribed angles intercept the same arc, then angles are congruent.

Since  $\angle A$  &  $\angle B$  both intercept arc CD, then  $\angle A \cong \angle B$ .



An angle inscribed in a semicircle is a right angle.

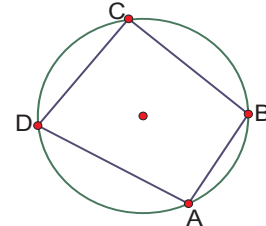
Since  $\widehat{MXN}$  is a  $180^\circ$ , its inscribed angle X must be  $90^\circ$ .



If a quadrilateral is inscribed in a circle, its opposite angles are supplementary.

$$m\angle A + m\angle C = 180^\circ$$

$$m\angle B + m\angle D = 180^\circ$$



1

$m\angle A = \underline{\hspace{2cm}}$

4

$x = \underline{\hspace{2cm}}$

$y = \underline{\hspace{2cm}}$

7

$x = \underline{\hspace{2cm}}$

$y = \underline{\hspace{2cm}}$

$z = \underline{\hspace{2cm}}$

2

$x = \underline{\hspace{2cm}}$

5

$x = \underline{\hspace{2cm}}$

$y = \underline{\hspace{2cm}}$

8

$x = \underline{\hspace{2cm}}$

$y = \underline{\hspace{2cm}}$

$z = \underline{\hspace{2cm}}$

3

$x = \underline{\hspace{2cm}}$

$y = \underline{\hspace{2cm}}$

$z = \underline{\hspace{2cm}}$

6

$x = \underline{\hspace{2cm}}$

$y = \underline{\hspace{2cm}}$

$z = \underline{\hspace{2cm}}$

9

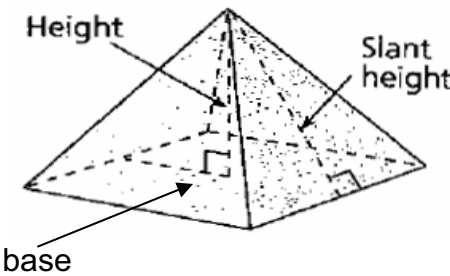
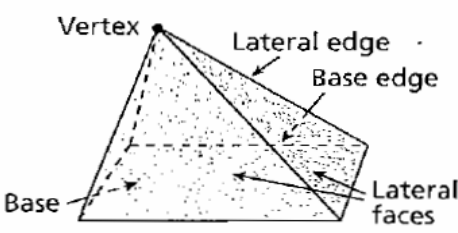
$x = \underline{\hspace{2cm}}$

$y = \underline{\hspace{2cm}}$

$z = \underline{\hspace{2cm}}$

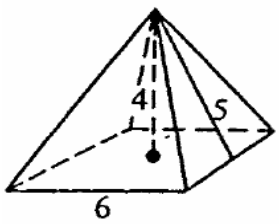
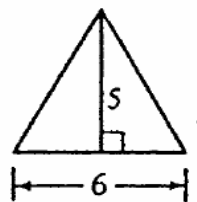
Solids I - Prisms and Pyramids

PYRAMIDS - Surface Area & Volume

  <p>*Notice the height and slant height form a right triangle with the base.</p>	<p><b>SURFACE AREA (SA) =</b> Sum of Areas of Faces</p> <hr/> <p><b>VOLUME (V) =</b> <math>\frac{1}{3} \times \text{Base Area (B)} \times \text{Height (h)}</math> <math>V = \frac{1}{3}Bh</math></p>
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Find the surface area and volume of the solid.

ex square based pyramid

5 is the slant height used in finding SA.

4 is the regular height used in finding V.

← One lateral face

To find SA, pull off one lateral triangle and find its area first.

$SA = 4(\text{area of side triangle}) + \text{base}$

$SA = 4(\frac{1}{2} \cdot 6 \cdot 5) + 6^2$

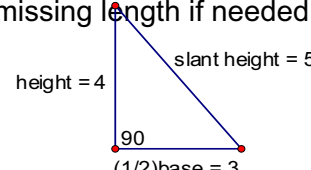
$SA = 60 + 36 = 96$

$V = \frac{1}{3} \cdot B \cdot h$

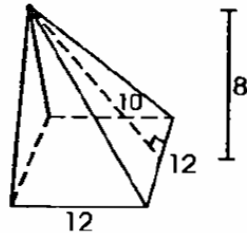
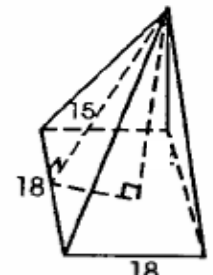
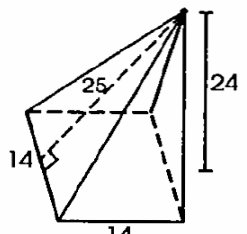
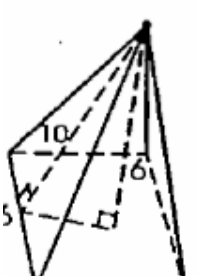
$V = \frac{1}{3} \cdot 6^2 \cdot 4$

$V = 48$

This relationship can be used to find a missing length if needed.



Find the surface area and volume of each square based pyramid.

<p>1</p>  <p>SA = _____ V = _____</p>	<p>3</p>  <p>SA = _____ V = _____</p> <p style="text-align: right; font-size: small;">*Use the special right triangle relationship to find the height.</p>
<p>2</p>  <p>SA = _____ V = _____</p>	<p>4</p>  <p>SA = _____ V = _____</p>